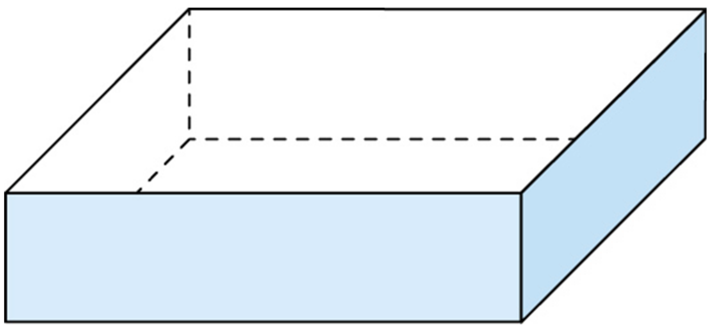
5 Calculus

Activity: What is optimization? (Teacher version)

Question

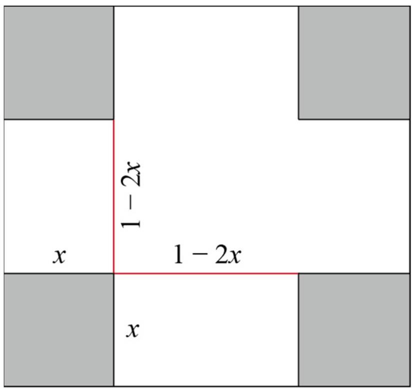
An open-topped box is formed by cutting congruent squares from the corners of a square piece of cardboard and turning up the edges. Before any alterations, the piece of cardboard has an area of 1 m^2.



Assuming you want the volume of the box to be the maximum it can possibly be, what size should the squares cut from each corner of the cardboard be?

Students must have knowledge of local maxima/minima before attempting.

Let x represent the length of a side of the square cut from a corner.



Express the volume V of the box as a function of x:

$V(x) = (1 - 2x)(1 - 2x)x$

$\quad\; = x - 4x^2 + 4x^3$

Finding $V'(x) = 1 - 8x + 12x^2$ and solving:

$1 - 8x + 12x^2 = 0$

$(1 - 6x)(1 - 2x) = 0$

$x = \frac{1}{6}$ m or $x = \frac{1}{2}$ m

We cannot cut $\frac{1}{2}$ m off each corner because the carboard is only 1 m square, so this leaves us with $\frac{1}{6}$ m.

Therefore, the maximum volume is when $x = \frac{1}{6}$ m. This gives a volume of:

$$V\left(\frac{1}{6}\right) = \left(1 - \frac{2}{6}\right)\left(1 - \frac{2}{6}\right)\frac{1}{6}$$

$$= \frac{2}{27}\text{ m}^3$$