2 Functions

Teaching support and guidance

Concepts

- Relationships
- Modelling
- Change

Outcomes

Students will learn how to create different representations of functions in order to model the relationships between variables. With the inquiry questions in mind, they will study how real-world relationships can be modelled.

Conceptual understandings

- Different representations facilitate modelling and interpretation of physical, social, economic and mathematical phenomena.
- Changing the parameters of a function changes the position, orientation and shape of the corresponding graph.

Inquiry questions

- Factual: How do we represent and use relationships?
- Conceptual: How do we manipulate functions?
- Debatable: What are the limitations of a model? How much can we change the parameters?

Factual: How do we represent and use relationships?

Concept: Relationships

Standard Level

Discussion: Straight-line geometry (S2.1)

Consider how a straight line is a representation of a relationship. What is it telling us? For example, in basic physics, the gradient of a distance–time graph tells us the speed of the object. Graphs are always telling a story.

Activity: Linear programming (S2.6)

During the first topic of straight-line geometry it is a nice add-on to demonstrate to the students the use of straight-line graphs in linear programming. It is a little tricky because it involves inequalities, but the application is worth it.

The following website contains guidance on how to graph linear inequalities:
www.mathsisfun.com/algebra/graphing-linear-inequalities.html

Have the students attempt the activity on linear programming. It includes a worked example and a separate question for them to attempt.

Discussion: Maximising (S2.5)

Talk the students through Worked Example 16.5 in the textbook. They can then attempt questions 5 and 6 from the following exercise.

To help students understand the concept of maximising, it will be useful for them to see the graph of the function. This can be done using their GDC or a graphing website. A quadratic model for the area of a rectangle can be a difficult concept for the students to grasp.

If the students need extra clarification about how area can be represented by a quadratic function, it can be useful to start with a square, which leads nicely to the idea of the power of two on the variables. The important fact is that, in the rectangle question, the dimensions must be defined using the same variable, which is the reason for the perimeter being given.

Conceptual: How do we manipulate functions?

Concepts: Change, Relationships

Standard Level

Discussion: Functions (S2.2)

Have the students consider the following TOK question while taking part in this discussion: Is mathematics simply the manipulation of symbols under a set of formal rules?

Before beginning the section on inverse functions, have the students think about the following:

Which came first, deriving the inverse of a function graphically or deriving it algebraically?

If mathematics is simply the manipulation of symbols, then one can assume that replacing y with x and then rearranging for y is simply 'the process'. This works in most cases and the vertical line test indicates if an inverse exists.

However, the vertical line test cannot be done before you have drawn the graph, so this leads us to believe that the graphing method must have been derived first?

Does the beauty and logic of mathematics become adapted into sets of formal rules that are to be followed? Can students identify any other areas where this dynamic exists?

An example could be $a^2 + b^2 = c^2$, which we know as Pythagoras' theorem. The original triangle often becomes ignored and the focus is put solely on the algebra.

Higher Level

PowerPoint: Composite functions (H2.7)

Using the PowerPoint, discuss with students the applications of composite and inverse functions and where they could potentially exist in real life.

The focus should be on the connection between numbers. The output from one function becomes the input to another. Emphasise the notation:

$$f(g(x))$$

When substituting a value for x we can see that it goes through the function g and then the output from this function becomes the input for f.

Therefore, the function $g(x)$ has changed from being the output to the input.

Activity: Functions transformations (H2.8)

Before teaching Section H2.8, as a starter activity, have the students attempt the exploration piece on transforming functions. It would be powerful for them to work in small groups to refine their explanations of what has happened to the function.

Debatable: What are the limitations of a model? How much can we change the parameters?

Concepts: Modelling

Standard Level

Activity: Bacterial growth (S2.6)

Modelling in mathematics is a difficult task. Students must appreciate that real-life mathematics does not always fit into a formula. The bacterial growth activity is a great example of this. Students can complete the task in small groups or individually. The task could also be done as a 'think–pair–share', where the students share their ideas after attempting it individually.

Students must learn to be critical of models and be able to identify limitations. For example, in the bacterial growth task there is no mention of the bacteria decreasing in number. Topics for discussion at this point include a vaccination, medicine, death rate (if talking about the spread of a virus).

How can we fit all of these limitations into our models? Is it possible? Or do we simplify our model to ensure the mathematics works for us?

Activity: Interpreting graphs (SL 2.6)

Using the task sheet provided, have the students analyse the graphs. It is very important that they consider the meaning of each calculation, as it can highlight the uncertainty of going beyond the data given.

When reviewing the answers with the class, discuss the validity of the models.

Extrapolation using models is very risky; when this takes place it is often referred to as 'uncharted territory'. There is an excellent article written by Tom Murphy entitled 'Ruthless Extrapolation': https://dothemath.ucsd.edu/2012/06/ruthless-extrapolation/ It is worth sharing with students, as it references various real-life examples, including TV shows, fossil fuel usage and the time it takes to cross the Atlantic Ocean.

Activity: Big wheel (S2.5, S2.6)

The Big wheel activity is a homework-style task. It is more structured than the bacterial growth task and offers scaffolding for identifying limitations to models.

The task should be introduced and attempted after the students have finished sinusoidal models towards the end of SL 2.5. However, given the scaffolding in the task, it will serve as a nice introduction to modelling a real-life situation (SL 2.6).

Emphasis should be put on the limitations of such a model. An idea for discussion (if not mentioned by a student) is: How do people disembark from the Ferris Wheel? This is a good time to discuss piecewise functions.