5 Calculus

Teaching support and guidance

Concepts

- Change
- Relationships
- Representation
- Validity′

Outcomes

Students will discover how calculus can be used to describe rates of change between two variables. Understanding these rates of change allows us to model, interpret and analyse real-world problems and situations.

Conceptual understandings

- Understanding the links between the derivative and the rate of change enables us to interpret the meaning of this in context.
- Optimization of a function shows us the largest or smallest value that a function can take in general and can be applied to a specific set of conditions to solve problems.

Inquiry questions

- Factual: What is a rate of change?
- Conceptual: What is optimization?
- Debatable: Do real-world problems fit a derivative model?
- Factual: How do we solve differential equations?

Factual: What is a rate of change?

Concepts: Change, Relationships

Standard Level

PowerPoint: What is a rate of change?

This PowerPoint builds on how rate of change builds on MYP work on gradients (the simplification of the process). This builds on prior knowledge of gradients and introduces the students to the notation $\frac{dy}{dx}$. It also brings in information from other subjects such as physics and use of 'rates of change' on displacement–time graphs and velocity–time graphs.

Debatable: Do real-world problems fit a derivative model?

Concepts: Relationships, Validity

Standard Level

Discussion: TOK: The relationships between mathematical models and physical reality (S5.3)

You might want to refer to question 13 in the Mixed Practice exercise of Chapter 9 of the SL student book in relation to this TOK link.

After students have finished attempting the question, ask:

- Is this a reliable model?
- What factors are the company maybe not considering?
- Is it possible to develop the perfect model for profit?

PowerPoint: Economics

This topic does blur the lines between SL and HL, especially when using applications of derivatives. You will have to talk about the concept of the second derivative in order to analyse graphs. You should emphasise to SL students that the second derivative will not be examined.

Links: Models in natural sciences and social sciences

Below are some links to information about some of the models mentioned in the *You are the Researcher* section of Chapter 9, Section 9A, of the SL student book:

- SIR model:
  www.maa.org/press/periodicals/loci/joma/the-sir-model-for-spread-of-disease-the-differential-equation-model
- Von Bertalanffy growth equation (more suited to HL):
  www.math.tamu.edu/~glahodny/Math147/Spring2017/Section%204.6.pdf
  www.pisces-conservation.com/growthhelp/index.html?von_bertalanffy.htm

- Black–Scholes formula (more suited to HL): www.investopedia.com/university/options-pricing/black-scholes-model.asp

Higher Level

PowerPoint: Kinematics (H5.13)

It is important that students recognise the connection between rate of change and calculus. When teaching basic gradients in lower grade levels, it is important to use terminology such as 'rate of change'. Kinematics is a practical example of how we can simplify the process of differentiation and integration.

This PowerPoint talks through the ideas behind kinematics and simplifies the logic in the calculations. The three-step process is a useful visual for students when they are attempting kinematics questions.

For example: if they are given the function for velocity. By using the visual aid, when they integrate the function, they will have a function for displacement. If they differentiate the function, they will have a function for acceleration.

You might also want to use this opportunity to address the following International Mindedness issue: Does the inclusion of kinematics as core mathematics reflect a particular cultural heritage? Who decides what is mathematics?

Conceptual: What is optimization?

Concepts: Representation, Change

Standard Level

Activity: What is optimization? (after S5.6)

Have students attempt the activity, which involves maximizing the volume of a box.

The following links contain some useful advice and examples relating to optimization:

- Leslie Chandrakantha of CUNY has simplified the process for us: http://archives.math.utk.edu/ICTCM/VOL23/C006/paper.pdf
- The website www.math24.net/optimization-problems-economics/ has some examples of financial optimization that you can use.

Factual: How do we solve differential equations?

Concepts: Representation, Change

Higher Level

PowerPoint: Euler's method in Excel (H5.16)

This is a PowerPoint that can be used to aid students in building an approximate solution to differential equations. Depending on the ability of your students you may want to share the included spreadsheet with them and let them change the variables. You may also want to build it with them using the PowerPoint. A video demonstration is also included in the resources.