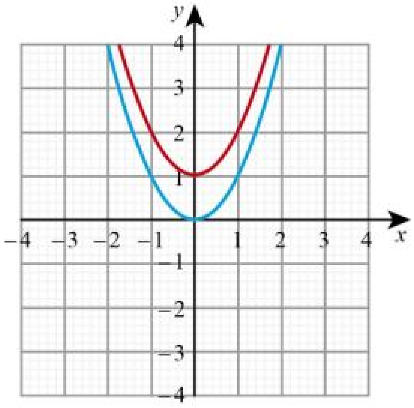
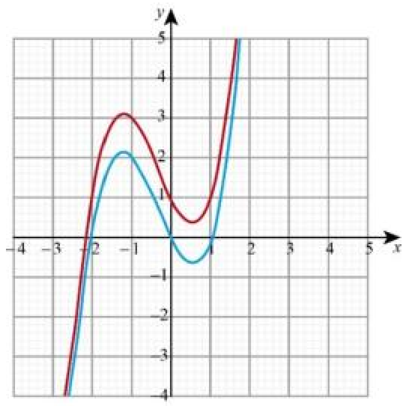
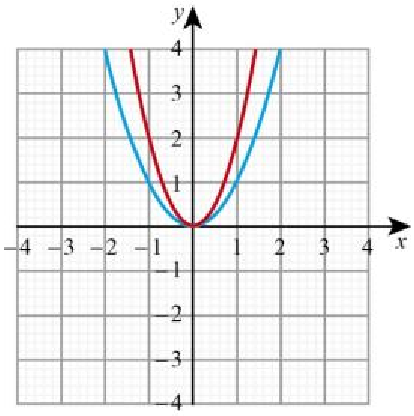
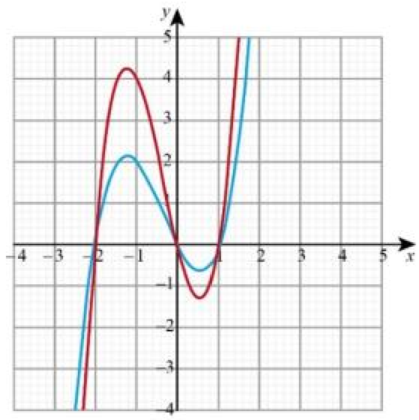
2 Functions

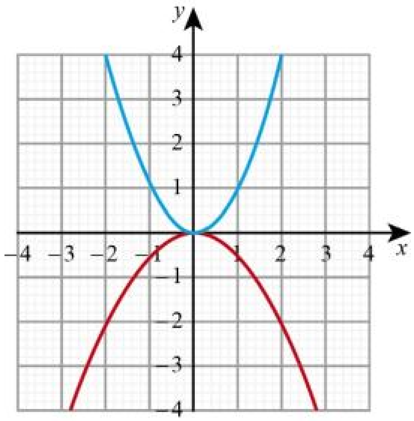
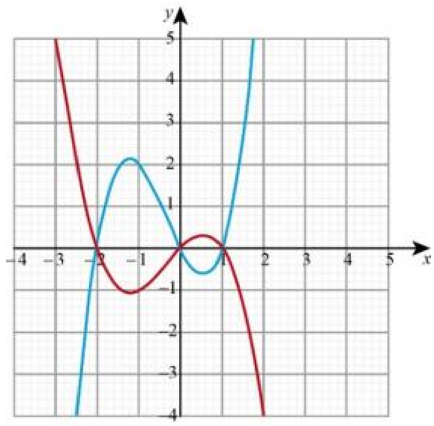
Activity: Transformations of functions (Teacher version)

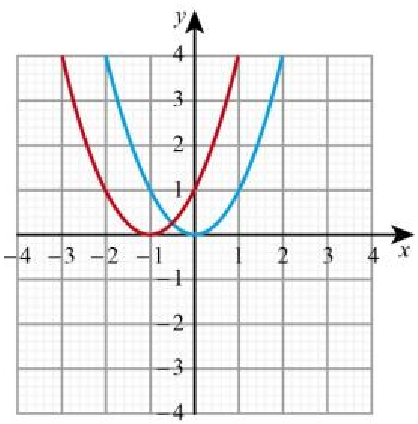
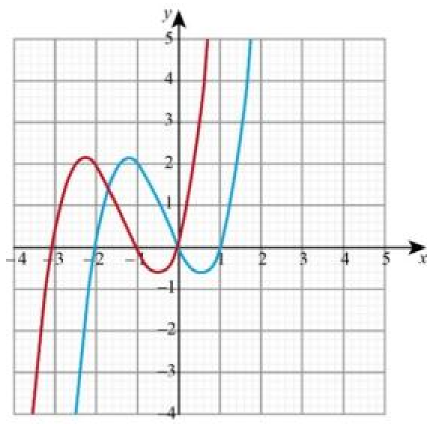
The original graphs of $f(x) = x^2$ and $g(x) = x(x-1)(x+2)$ are shown in each graph below.

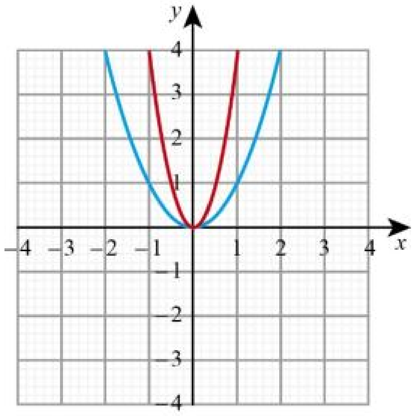
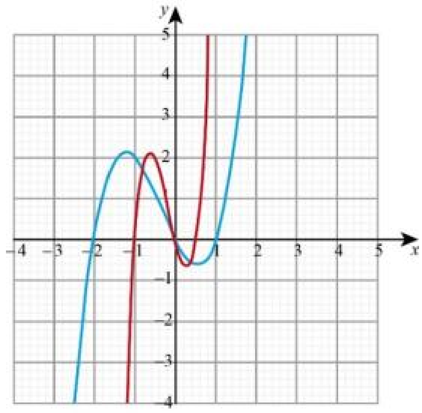
- On the same set of axes, sketch BY HAND (without your GDC) the transformed graphs of $y = f(x)$ and $y = g(x)$ given in the left-hand column.
- Label the coordinates of any important points.
- Check to see if your sketches are correct by CHECKING on your GDC (again, try to think about it and do it by hand at first).
- Describe and explain the general transformation in the row below.

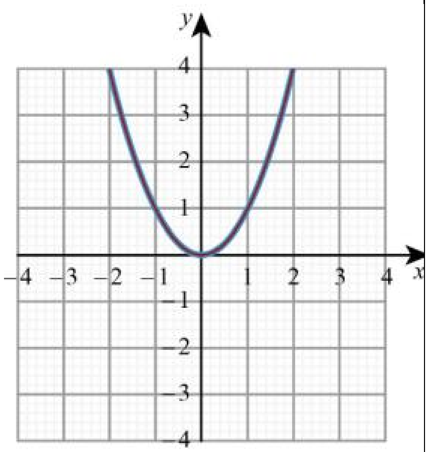
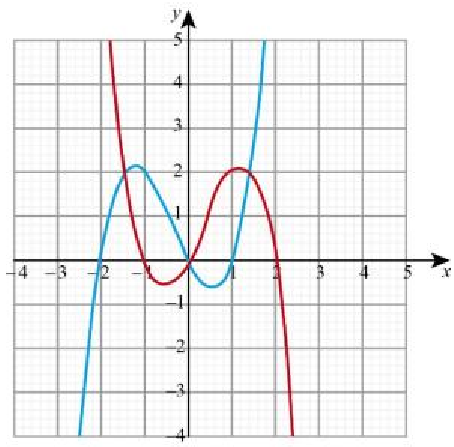
Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
$y = \mathrm{f}(x) + 1$ $y = \mathrm{g}(x) + 1$		
Description of general transformation with a brief explanation	$y = \mathrm{f}(x) + c$ translates the graph c-units vertically on the y-axis because all the y-values are changed by c.	

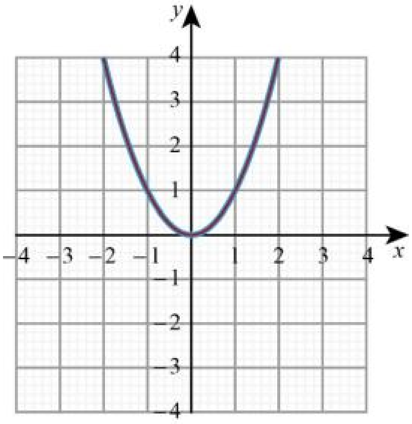
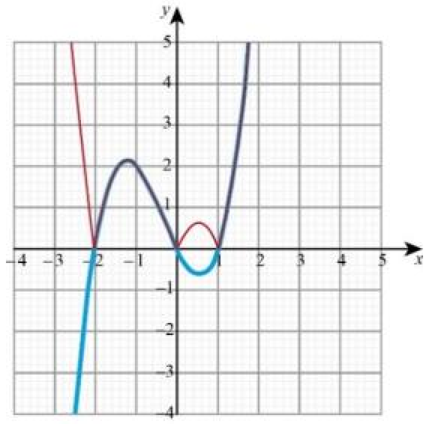
Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
$y = 2\mathrm{f}(x)$ $y = 2\mathrm{g}(x)$		
Description of general transformation with a brief explanation	$y = a\mathrm{f}(x)$ vertically dilates (stretches) the graph of $y = \mathrm{f}(x)$ by a scale factor of a.	

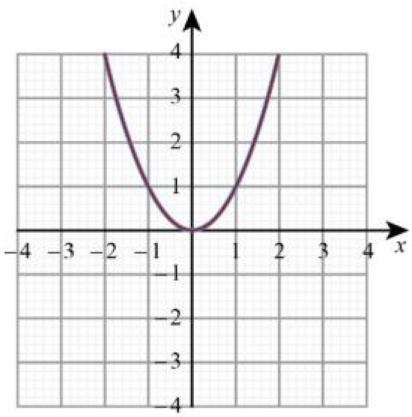
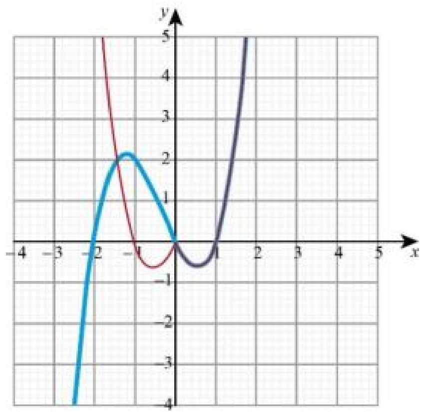
Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
$y = -\frac{1}{2}\mathrm{f}(x)$ $y = -\frac{1}{2}\mathrm{g}(x)$		
Description of general transformation with a brief explanation	$y = a\mathrm{f}(x)$ vertically dilates (stretches) the graph of $y = \mathrm{f}(x)$ by a scale factor of a. (NOTE: for this graph, the stretch factor is $\frac{1}{2}$, which is actually a vertical compression.) If $a < 0$, graph is also reflected in the x-axis.	

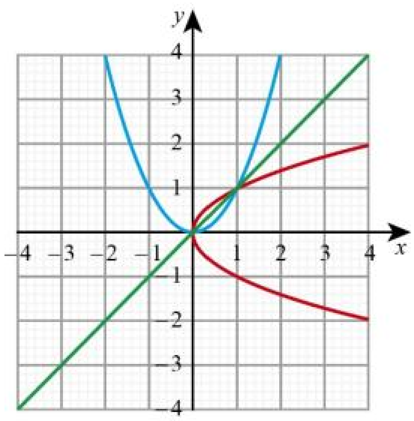
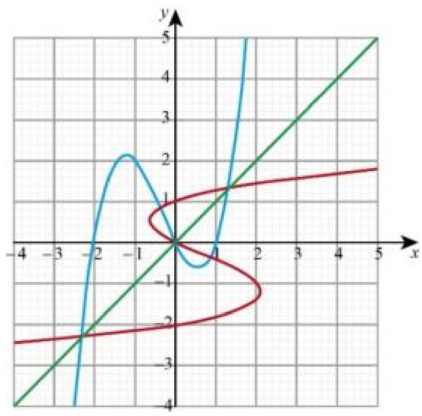
Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
$y = \mathrm{f}(x + 1)$ $y = \mathrm{g}(x + 1)$		
Description of general transformation with a brief explanation	For $y = \mathrm{f}(x + c)$, the graph is translated horizontally '$-c$' units. This relates directly to all x-intercepts. The reason is that if you plot an (x, y) point, you use the value not from $y = \mathrm{f}(x)$ but rather from $y = \mathrm{f}(x + c)$ and plot $(x, \mathrm{f}(x + c))$.	

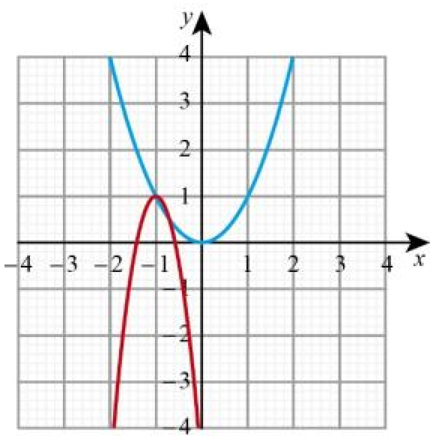
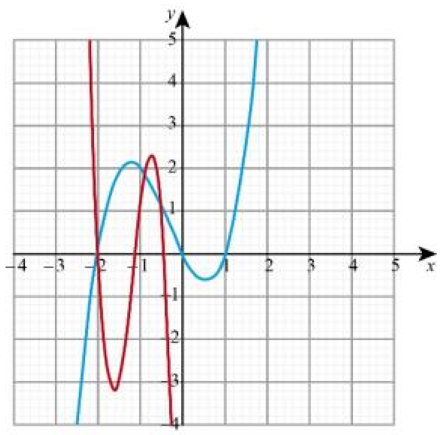
Specific transformation description	Transformed graph of $y = f(x)$	Transformed graph of $y = g(x)$
$y = f(2x)$ $y = g(2x)$		
Description of general transformation with a brief explanation	For $y = f(bx)$, the graph is stretched horizontally by scale factor $\frac{1}{b}$. The reason is that if you plot an (x, y) point, you use the value not from $y = f(x)$ but rather from $y = f(bx)$ and plot $(x, f(bx))$.	

Specific transformation description	Transformed graph of $y = f(x)$	Transformed graph of $y = g(x)$
$y = f(-x)$ $y = g(-x)$		
Description of general transformation with a brief explanation	For $y = (bx)$ when $b < 0$, the graph is reflected in the y-axis. The reason is that if you plot an (x, y) point, you use the value not from $y = f(x)$ but rather from $y = f(-x)$, which is on the opposite side of the y-axis, and plot $(x, f(-x))$.	

Specific transformation description	Transformed graph of y = f(x)	Transformed graph of y = g(x)
$y = \lvert f(x) \rvert$ $y = \lvert g(x) \rvert$		
Description of general transformation with a brief explanation	For $y = \lvert f(x) \rvert$, the graph is the same when $f(x) > 0$ and is reflected in the x-axis when $f(x) < 0$. This is because all of the y-values which were negative are now positive (since it is the absolute value).	

Specific transformation description	Transformed graph of y = f(x)	Transformed graph of y = g(x)
CHALLENGE: $y = f(\lvert x \rvert)$ $y = g(\lvert x \rvert)$		
Description of general transformation with a brief explanation	For $y = f(\lvert x \rvert)$, the graph when $x > 0$ is unchanged. However, when $x < 0$, the graph when $x > 0$ is reflected in the y-axis. The reason is that if you plot an (x, y) point, you use the value not from $y = f(x)$ but rather from $y = f(-x)$, which is on the opposite side of the y-axis, and plot $(x, f(-x))$.	

Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
CHALLENGE: $x = \mathrm{f}(y)$ $x = \mathrm{g}(y)$		
Description of general transformation with a brief explanation	This is simply the inverse of $y = \mathrm{f}(x)$, which is a reflection in the line $y = x$.	

Specific transformation description	Transformed graph of $y = \mathrm{f}(x)$	Transformed graph of $y = \mathrm{g}(x)$
PUTTING IT ALL TOGETHER: $y = -2\mathrm{f}(2(x+1))+1$ $y = -2\mathrm{g}(2(x+1))+1$		
Description of general transformation with a brief explanation	Following the order of transformations: 1 Vertical dilation by scale factor 2 and a reflection in the x-axis 2 Horizontal dilation by scale factor $\frac{1}{2}$ 3 Horizontal translation –1 units and vertical translation +1 units.	