1 Number and algebra

Teaching support and guidance

Concepts

- Relationships
- Equivalence
- Patterns
- Representation
- Modelling

Outcomes

In this unit, students will study how number and algebra allow us to represent patterns, show equivalencies and discover relationships in different situations. This will enable them to model real-world mathematics.

With the inquiry questions in mind, they will study how numbers are represented, how they are combined and how quantities change in different situations.

Students will understand the relationships between different quantities and patterns in number, finance, and sequences and series.

Conceptual understandings

- Number and algebra allow us to represent patterns, show equivalencies and discover relationships that enable us to model real-life situations.
- Algebra is an abstraction of numerical concepts and employs variables to solve mathematical problems.

Inquiry questions

- Factual: What relationships exist in mathematics?
- Debatable: How does a quantity grow?
- Debatable: How have technological advances affected the nature and practice of mathematics?
- Conceptual: Are matrices evidence for a simple underlying mathematical reality?

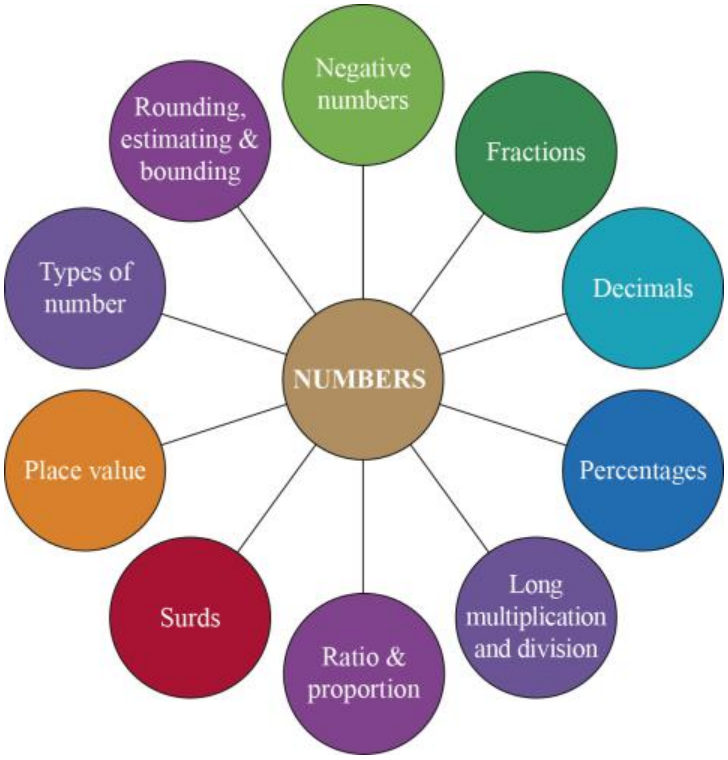
Factual: What relationships exist in mathematics?

Concept: Relationships, Equivalence, Patterns

Standard Level

Discussion: Introduction to equivalence (S1.1)

When starting the unit, it is a good idea to gauge students' understanding of equivalences. Have them list as many as they can think of within mathematics. A mind map may be a nice activity in small groups:



How do we calculate 10% of a number? We multiply by 0.1. This is a simple relationship that exists to make use of multipliers with percentage amounts.

If we want to increase a number by 10%, we multiply by 1.1. This is how basic relationships in mathematics enable us to simplify a process.

Discussion: The form $a \times 10^k$ (S1.1)

Why does the form $a \times 10^k$ exist? Here is a link to a YouTube clip with a quick example: www.youtube.com/watch?v=Sybsv4MELE8

Have students think about the following and how using this form of the numbers can make the calculations easier.

The mass of the Earth is about 5 980 000 000 000 000 000 000 000 kg.

What is the mass of the Earth using scientific notation?

If the average person weighs 60 kg and the population of the Earth is approximately 7 billion people, how many times heavier is the Earth than the entire population of people on the Earth?

Activity: Large numbers (S1.1)

This task is designed as a consolidation of students' knowledge of scientific notation. They will have seen this notation before but maybe not have used the applications of it. Questions 9 to 15 will challenge HL students to apply their knowledge and develop problem-solving skills.

PowerPoint: The Fibonacci numbers and the golden ratio (S1.1)

Patterns and relationships can appear in many different ways. What is obvious to one person may not be clear to another. This PowerPoint introduces the Fibonacci numbers and the golden ratio, and includes links to the following videos:

- The magic of Fibonacci numbers – a TED talk by mathematician Arthur Benjamin: www.youtube.com/watch?v=SjSHVDfXHQ4
- Places you won't believe the golden ratio and Fibonacci sequence appears – a look at some of the many natural occurrences and practical applications of Fibonacci numbers and the golden ratio: www.youtube.com/watch?time_continue=8&v=RqqErDSLtwE

This PowerPoint addresses the TOK question: Is all knowledge concerned with the identification and use of patterns?

Link: Patterns in Pascal's triangle (S1.1)

As an extra activity, that will benefit the students for future topics, such as binomial expansions or combinations, students can explore the patterns within Pascal's triangle.

There are some nice examples on the website www.mathsisfun.com/pascals-triangle.html

Debatable: How does a quantity grow?

Concepts: Representation, Relationships, Equivalence

Standard Level

PowerPoint: Allowance payments problem (S1.2, S1.3)

While studying the sections on arithmetic and geometric sequences and series, it is good to ask the students the question in this PowerPoint as a starter activity. It can be attempted in groups or individually. A big piece of A3 paper is a good tool for collaboration.

This is an excellent example of how quickly an amount of money can grow and most students will (hopefully) choose Option 1 because they will presume it will be the larger amount of money. In fact, Option 2 yields a much higher amount.

Have the students calculate the amount they receive for both options. For accuracy they may use a GDC or a spreadsheet. An Excel spreadsheet showing both options is included in the resources.

Discuss with students the concept of 'representation' – how does using technology help us with a question like this?

	Option 1			Option 2	
				Dollars	Total
Days:	731		Day 1	$ 0.01	$ 0.01
Total:	$731,000.00		Day 2	$ 0.02	$ 0.03
			Day 3	$ 0.04	$ 0.07
			Day 4	$ 0.08	$ 0.15
			Day 5	$ 0.16	$ 0.31
			Day 6	$ 0.32	$ 0.63
			Day 7	$ 0.64	$ 1.27
			Day 8	$ 1.28	$ 2.55
			Day 9	$ 2.56	$ 5.11
			Day 10	$ 5.12	$ 10.23
			Day 11	$ 10.24	$ 20.47
			Day 12	$ 20.48	$ 40.95
			Day 13	$ 40.96	$ 81.91
			Day 14	$ 81.92	$ 163.83
			Day 15	$ 163.84	$ 327.67
			Day 16	$ 327.68	$ 655.35
			Day 17	$ 655.36	$ 1,310.71
			Day 18	$ 1,310.72	$ 2,621.43
			Day 19	$ 2,621.44	$ 5,242.87
			Day 20	$ 5,242.88	$ 10,485.75
			Day 21	$ 10,485.76	$ 20,971.51
			Day 22	$ 20,971.52	$ 41,943.03
			Day 23	$ 41,943.04	$ 83,886.07
			Day 24	$ 83,886.08	$ 167,772.15
			Day 25	$ 167,772.16	$ 335,544.31
			Day 26	$ 335,544.32	$ 671,088.63
			Day 27	$ 671,088.64	$ 1,342,177.27
			Day 28	$ 1,342,177.28	$ 2,684,354.55
			Day 29	$ 2,684,354.56	$ 5,368,709.11
			Day 30	$ 5,368,709.12	$ 10,737,418.23

Option 1 is an example of an arithmetic series and Option 2 is an example of a geometric series. This demonstrates the power of multiplying by a ratio compared to adding a common difference.

If students build a spreadsheet model, have them use formulae and try different starting amounts. How much money do you have to receive each day in Option 1 to earn the same as in Option 2?

PowerPoint: Credit cards (after S1.4)

This PowerPoint provides an introduction to the dangers of credit and walks students through an example of how interest can accumulate. An Excel spreadsheet showing the calculations for this example is included in the resources.

Activity: The mathematics of credit (after S1.4)

This activity requires students to find a real-life credit card that is being offered and to use the advertised rate to calculate how much interest they might accumulate if they were to use it. Have the students grade their own piece of work using the assessment criteria provided. If you have time, it can be powerful to have students grade each other's tasks.

PowerPoint: Logarithms in chemistry (S1.5)

The PowerPoint discusses the application of logarithms in relation to pH value in chemistry. It offers a good discussion point as to what prior knowledge the students have in relation to logarithms.

Debatable: How have technological advances affected the nature and practice of mathematics?

Concepts: Relationships

Standard Level

Discussion: How have technological advances affected the nature and practice of mathematics? (S1.4)

Consider the following TOK question: How have technological advances affected the nature and practice of mathematics?

Conrad Wolfram gives a TED talk on 'Stop teaching calculating, start teaching math'. The talk is around 20 minutes long, so could serve as a homework task for discussion in class during the next lesson:

http://blog.wolfram.com/2010/11/23/conrad-wolframs-ted-talk-stop-teaching-calculating-start-teaching-math/

After watching the video, consider the question regarding allowance payments. How time-consuming would this have been without the use of a spreadsheet? Could students have gauged the concept by estimating without technology? Would the calculations have hampered the understanding?

When attempting questions on the topic of finance, encourage students to write down the processes they are following on their GDC, rather than the actual numbers and results. Students can sometimes become lost when inputting numbers into their GDC and find it difficult to retrace their steps.

Conceptual: Are matrices evidence for a simple underlying mathematical reality?

Concepts: Relationships, Modelling

Higher Level

Discussion: Matrices (H1.14)

Consider the following questions from a TOK perspective: Are matrices evidence for a simple underlying mathematical reality? What are the relationships between mathematics, sense, perception and reason? If we can find solutions of higher dimensions can we reason that these spaces exist beyond our sense perception?

Links: Using matrices for coding (H1.14)

By using the examples at the following link, attempt the exercises at the end of the activities: http://crypto.interactive-maths.com/hill-cipher.html. Alternatively, if you have problems running Adobe Flash try this site instead:

http://polymathprogrammer.com/2008/08/11/matrices-for-programmers/. Students must have completed the matrices sections H1.14 and H1.15 before attempting these exercises.

This YouTube clip also demonstrates coding using matrices: www.youtube.com/watch?v=h7dwHg3EZjE

It would be a powerful exercise for the students to code and decode their own messages using matrices. The outcome of this activity would be an appreciation of the amount of patience it takes and how automated machines can make a process like this seem very simple.

For those students who are interested, there are resources that discuss the use of matrices in computer graphics.

Here are some links for discussion:

- Matrices in computer graphics https://sites.math.washington.edu/~king/coursedir/m308a01/Projects/m308a01-pdf/yip.pdf
- Behold the beast! The magnificent 2D matrix! https://ncase.me/matrix/

PowerPoint: Eigenvectors and PageRank (after H1.15)

The PowerPoint discusses the use of matrices in web page ranking. It talks through a basic scenario involving four web pages. The students will need to have covered eigenvector and eigenvalue calculations before viewing the presentation.

Although quite complicated, there are many further applications of eigenvalues and eigenvectors that the students could investigate and which could form the basis of their exploration. For example, they might want to watch the video at the link below of the Tacoma Narrows bridge collapse and explore how eigenvalues could have been used to prevent this from happening.

- YouTube video of the Tacoma Narrows bridge collapse www.youtube.com/watch?v=j-zczJXSxnw