3 Geometry and trigonometry

Teaching support and guidance

Concepts

- Modelling
- Space
- Systems
- Representation

Outcomes

Students will understand how we use geometry and trigonometry to quantify the physical world, enhancing their spatial awareness in two and three dimensions.

Conceptual understandings

- Different representations of trigonometric expressions help to simplify calculations.
- Graph theory algorithms allow us to model complex real-world problems.

Inquiry questions

- Factual: How is trigonometry used in the physical world?
- Conceptual: How does mathematics model real life?
- Factual: How do we know that the Earth is round?
- Debatable: Which is the better measure of an angle: degrees or radians?

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Factual: How is trigonometry used in the physical world?

Concepts: Space, Modelling

Standard Level

Link: How does surveying work? (S3.1)

The following link takes you to a video that introduces students to field book entries. This is a vital part of surveying and forms the basis for calculations involving distances and areas: www.youtube.com/watch?v=2_edNawzsUM

Please see the following link for some example calculations: www.youtube.com/watch?v=DNXzcOp9kM8

PowerPoint: Heron's formula for the area of a triangle (S3.2)

Before finishing SL 3.2, this is an interesting side-note on a further calculation for the area of a triangle.

The PowerPoint gives information on a formula related to surveying: Heron's formula. Here is a link to a website that explains how the formula is obtained and gives some example calculations: https://brilliant.org/wiki/herons-formula/ (Note that although the PowerPoint only considers a right-angled triangle, Heron's formula applies to any triangle.)

Factual: How do we know that the Earth is round?

Concept: Space

Standard Level

PowerPoint: Calculating the distance between two points on Earth (S3.3, S3.4)

International mindedness: The following link describes the use of triangulation to find the curvature of the Earth in order to settle a dispute between England and France over Newton's theory of gravity: www.popularmechanics.com/science/environment/a6674/the-remarkable-story-of-the-first-accurate-measure-of-the-earth/

The PowerPoint talks through the more difficult calculation involving the Earth's 'small circles' (i.e. two places with the same latitudinal position).

Activity: Distances on Earth (S3.3, S3.4)

This activity gives students a chance to apply the trigonometry described in the PowerPoint.

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Conceptual: How does mathematics model real-life?

Concepts: Modelling, Systems, Space

Standard Level

PowerPoint: Voronoi diagrams (S3.6)

The PowerPoint brings together the ideas listed below and offers some examples of what Voronoi diagrams are used for.

Students can attempt their own Voronoi diagram using the step-by-step instructions described by the video linked to in the PowerPoint: www.instructables.com/id/Hand-Drawn-Voronoi-Diagrams/

There is also a nice applet that students can use to generate a quick Voronoi diagram: http://alexbeutel.com/webgl/voronoi.html

Here is a link to a PDF file from Texas Instruments that acts as an activity based around Voronoi diagrams, inspired by an episode of the crime-drama television series *Numb3rs*: https://education.ti.com/~/media/324A760B278742029CD938F553974ADE

Higher Level

Activity: Seven Bridges of Königsberg (HL 3.15)

The activity begins by introducing the class to the Bridges of Königsberg problem. It then outlines some concepts of graph theory before leading students through a series of networks.

PowerPoint: The four colour theorem (H3.15)

The PowerPoint talks through the simplicity of the four colour theorem. There are printouts for the students to attempt.

Have students think about the following TOK question, which is addressed in the final slide of the PowerPoint: If a theorem is proved by computer, how can we claim to know that it is true?

Debatable: Which is the better measure of an angle: degrees or radians?

Concept: Representation

Higher Level

Activity: Unit circle (H3.8)

Have the students attempt the investigation in order to generate the sine, cosine and tangent curves by using the coordinates from the unit circle. Emphasise how the systems can be manipulated using the transformation of graphs knowledge from the previous unit.

It is also important to emphasise the use of transforming graphs from Unit 2 when answering the questions on the task.

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There are a few good applets for the unit circle online:

- Geogebra: www.geogebra.org/m/nv9vex3X
- Maths is Fun: www.mathsisfun.com/algebra/trig-interactive-unit-circle.html

The Maths is Fun website offers the graphs of the trigonometric functions alongside the unit circle. This is useful for understanding how the graphs are generated.

Activity: Trigonometric (circular) graphs (H3.8)

This short activity provides students with another opportunity to check they have grasped the concepts explored in the previous activity.

PowerPoint: Radians (H3.7)

Immediately after you have completed the unit circle investigation, talk to the students about radian measures.

Using the PowerPoint, discuss with students the need for radian measures and how we convert between degrees and radians. The emphasis should be on the fact that in order to draw a scale version of each trigonometric function we need to have different units on the x- and y-axes. The need for radians is so that we can use the same units on both sets of axes. The features of the graph must exist in the same space. There are images to show the differences in appearance when using degrees and radians.

Extension activity: Fourier waves (H3.8)

The Fourier waves extension task is designed to give students an extra application of the sine and cosine graphs. It may form the basis for an exploration piece.

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